TIDE TABLES

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The tidal knowledge of the ancients appears to have been very meager; in fact, little mention of the tide is found in their literature. The chief maritime peoples of antiquity, living on or near the Mediterranean where the tide is small, could disregard it for practical purposes, and it likewise attracted little attention as a theoretical study. And, as with early explanations of other physical phenomena, the earlier attempts at explaining the tides were based largely on fanciful notions, such as the breathing of the earth and the power exerted by supernatural beings.

More rational theories ascribed the tide to differences in the level of the sea, to the discharge of rivers into the sea, to whirlpools and eddies, and finally to sun and moon. We have record of the fact that more than three centuries before the beginning of the Christian era the relationship between moon and tide had been noted. This relationship, however, received no rational explanation until the latter half of the seventeenth century when Newton formulated the law of gravitation and proved that the tides were one of its necessary consequences.

The ancients had little or no need for tide tables, and even until very recent times the navigator troubled himself little about accurate tidal data, for ships were operated on a schedule subject largely to the saving clause "weather and tide permitting." Now, however, an advance knowledge of the state of the tide is of great importance in maintaining an exact schedule at many places, since a change in the depth over the bar of but a few feet may mean the difference between getting a leviathan into port immediately or being compelled to wait several hours for a favorable tide.

Accurate tide tables became of prime importance during the World War; for in waiting for a favorable tide to enter port a vessel was exposed not only to possible bad weather but also to the more serious hazards of submarine attack. During the World War the importance of accurate tidal predictions was made manifest in other ways. In the case of a vessel being chased the navigator who had at hand accurate tidal data could decide whether a short cut over a shoal might be risked. During the bombardment of the Belgian coast the Allied war vessels were exposed to torpedo attack from the sea. By anchoring in the shoalest water possible considerable security was afforded, since if a torpedo was fired it was more likely to strike the shoal than the ship. That accurate tidal data were requisite for this purpose is evident from the fact that on the Belgian coast there is a range of from ten to sixteen feet in the rise and fall of the tides.

The oldest tide table of which there appears to be any record is one now

in the library of the British Museum. It is a manuscript table, apparently written in the thirteenth century, and gives the time of "flod at london brigge"—the time of high water at London Bridge. The modern navigator would scarcely recognize this as a tide table, for the times of tide are given not for calendar days of the month but only with reference to the moon's age. Of the modern tide tables the first appear to have been published by the British Admiralty in 1833. These tide tables gave the predicted times of high water only. In 1839 the French Hydrographic Service began the issue of its "Annuaire des Marées," and in 1853 the U. S. Coast and Geodetic Survey published its first tide tables.

Each of the leading maritime nations now publishes tide tables annually, a year or more in advance, for the use of its navy and merchant marine. For our own country this important work devolves upon the Coast and Geodetic Survey which early in May of every year issues the tide tables for the whole of the following year. These tide tables now cover the entire maritime world and give for every day of the year the predicted time and height of high and low water at eighty-one of the more important ports of the world. In addition these tables also contain sufficient tidal data for some thirty-five hundred secondary ports, which, in connection with the daily predictions given for the principal ports, enable the American navigator to make use of any port in the world.

Underlying Principles

The tides, as is well known, are due to the gravitational action of sun and moon upon the rotating earth. The moon is the principal agent, or as the popular expression has it, "the tide follows the moon." This is due to the fact that the tide-generating force of a body varies directly as its mass and inversely as the cube of its distance from the earth. The sun has a mass almost 26,000,000 times as great as that of the moon, but because its distance from the earth is 389 times that of the moon's, its tide-producing power is to that of the moon as 26,000,000 is to $(389)^3$ or somewhat less than half.

While the mathematical expression for the tide-generating forces of the sun and moon may be derived without difficulty, the tides as they actually occur in nature have been so profoundly modified by terrestrial features as to bear little or no resemblance to the purely theoretical tide. In other words, while the tidal *forces* for any point on the earth may be computed from a general formula based on astronomical considerations, it is not possible to derive such a general formula for determining the time and height of tide. The tides at places but a short distance apart may be very dissimilar. To take but a single example: at the Atlantic end of the Panama Canal the tides have a rise and fall of about one foot, and at certain times of the month but one high and one low water occur during a day; at the Pacific end the rise and fall is from twelve to sixteen feet, and every day

there are two high and two low waters. The tides, then, are local phenomena, and the first step in the making of a tide table for any given port is to secure tidal observations at that port for a period varying from a month to a year or more. These observations may then be treated in two different ways known technically as the nonharmonic and harmonic methods. The former method is the simplest and the one used in the making of the earlier tide tables.

The Nonharmonic Method

This method is based on the close relationship existing between the time of tide at any given place and the moon's meridian passage. It begins by determining, from observations made at the port for which predictions are desired, the intervals elapsing between the moon's meridian passage and the times of tide. These time intervals, known respectively as the high-water and the low-water lunitidal intervals, have an approximately constant value for any given place and after having been once determined, say from a month or more of observations, may be used for making a rough tide table for that place by adding to the times of the moon's meridian passage as given in a nautical almanac.

The lunitidal intervals at any given place, as stated above, are only approximately constant. During a lunar month they undergo periodic changes, depending principally on the phase and declination of the moon. From long series of tidal observations these periodic changes may be determined; and by using these as corrections to the lunitidal intervals satisfactory predictions may be secured. It was by various modifications of the method as above outlined, with corrections for inequalities in the motions of sun and moon, that tide tables were computed for many years.

The heights of high and low water were predicted in a similar manner. The average heights for high water and for low water at the port for which predictions were desired were determined from observations. To the average heights corrections were then applied for changes in the phase and parallax of the moon, these corrections likewise being derived from observations. The tide tables produced in this way, although only approximately correct, worked quite satisfactorily for the Atlantic coast of the United States and for Europe. However, when applied to the prediction of tides of a different type, such as are found in the Pacific and Indian Oceans, the nonharmonic method necessitated so many corrections as to become prohibitive.

Types of Tide

The different varieties of tides occurring in nature may be divided into three types: semidaily, daily, and mixed. In the semidaily type, as its name indicates, there are two high and two low waters each day, high water and low water following each other at intervals of approximately six hours, the morning and afternoon tides being similar. To this type belong the tides on the Atlantic coasts of the United States and Europe.

At many places—as for instance: St. Michael, Alaska; Manila, Philippine Islands; Batavia, Java—we find a totally different type of tide. Here high water and low water will frequently be more than twelve hours apart; or, in other words, instead of two high and two low waters in a day, as on the Atlantic coast, there will be but one high and one low water in a day. To this type of tide the name "daily" has been given.

As a matter of fact, there is no place known where the tide is always of the daily type. At all places where this type predominates periods of one

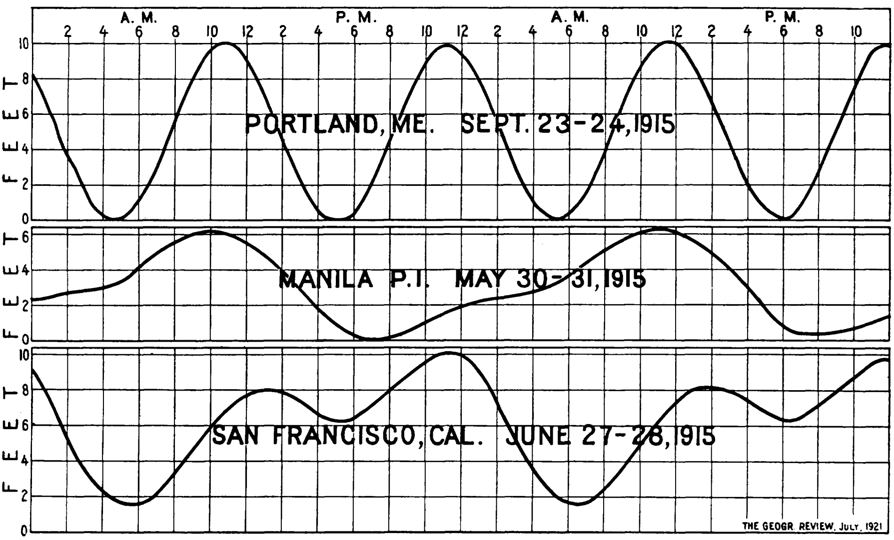

FIG. 1—Types of tide.

or more days occur during which there are two high and two low waters each day, these periods coming about the time that the moon's declination is changing from north to south or south to north. But, though two high and two low waters occur, they differ strikingly from the semidaily type in that the morning and afternoon tides are unlike both in extent and in duration of rise and fall. Such tides, having two high and two low waters each day but with morning and afternoon tides differing considerably, are known as the mixed type. This is the prevailing type on the coasts washed by the Pacific and Indian Oceans.

In the accompanying diagram the three types of tide are illustrated by plottings of hourly heights of the tide as observed for a period of two days in 1915 at Portland, Me., Manila, P. I., and San Francisco, Cal.

The tide curve for Portland exemplifies the semidaily type of tide. Beginning with the low water about 5 A. M., September 23, the water rises for a period of about six hours, falls for a like period, and repeats the cycle again in the afternoon and similar cycles the next day. In each case the rise or fall is about ten feet and takes place during an interval of about

six hours; in other words, both in extent and in duration of rise and fall the morning and afternoon tides are similar; and this is typical of the semidaily tide.

At Manila the tide is predominantly of the daily type, that is during the greater part of the time but one high water and one low water occur daily. Thus on May 30, 1915, high water occurred about 10 A. M., low water about 7 P. M.; and the next day likewise there occurred but one high and one low water.

The mixed type is illustrated by the tide at San Francisco for June 27 and 28, 1915. On both days we have two high and two low waters, but there is a striking difference between morning low water and afternoon low water and between morning high water and afternoon high water. This difference between morning and afternoon tides is of a twofold character: the extent of rise or fall is unlike and the duration of rise or fall is unequal. This difference between the two tidal cycles of the day is what distinguishes the mixed type of tide from the semidaily type.

For predicting the semidaily type of tide the nonharmonic method, as previously stated, gives satisfactory results. When applied to the prediction of the mixed type of tide this method, even where successful, necessitates an increase in the number of corrections to give passable results; but when tried for ports where the tide frequently becomes daily it falls down badly. Before the need for accurate tide tables covering the whole maritime world became pressing, a more effective method of treating and predicting tides was introduced. This, known as the harmonic method, is now used in the preparation of the tide tables issued by the United States government.

The Harmonic Method

If the motion of the moon around the earth and that of the earth around the sun could be made to take place in circular orbits in the plane of the equator with earth and sun as centers respectively, and, furthermore, if the lunar month consisted of an integral number of solar days, tidal prediction would be a very simple matter. It would only be necessary to observe the tides at any given port for a lunar month, and these observations would then constitute a very exact tide table for that port for the future; for the tide of any day of a lunar month would be exactly similar to the tide of the same day of any other lunar month. The actual motions of sun and moon are complicated; they can, however, be resolved into simple components; and this is what is done by the harmonic method. It substitutes for the sun and moon as tide-producing agencies a number of simple hypothetical bodies which, with respect to the earth, have circular orbits in the plane of the equator. Each of these tide-producing bodies is assumed to give rise to a tide of its own kind, and the tide as it actually occurs in nature is thus considered as being made up of a number of simple tides each of which has a period corresponding to the period of its particular, hypothetical tidal body.

The periods of revolution of the assumed tidal bodies, and therefore the periods of the simple constituent tides, are determined by astronomical calculations from the known motions of the moon and sun. These periods being independent of local conditions are therefore the same for all places on the surface of the earth; what remains to be determined for the various simple constituent tides is their phases and amplitudes, which vary from place to place according to type, time, and range of tide. The mathematical process by which these phases and amplitudes are disentangled from tidal observations is the very ingenious method known as harmonic analysis and is due to the brilliant and versatile physicist, William Thomson, better known as Lord Kelvin, who first proposed it in 1867.

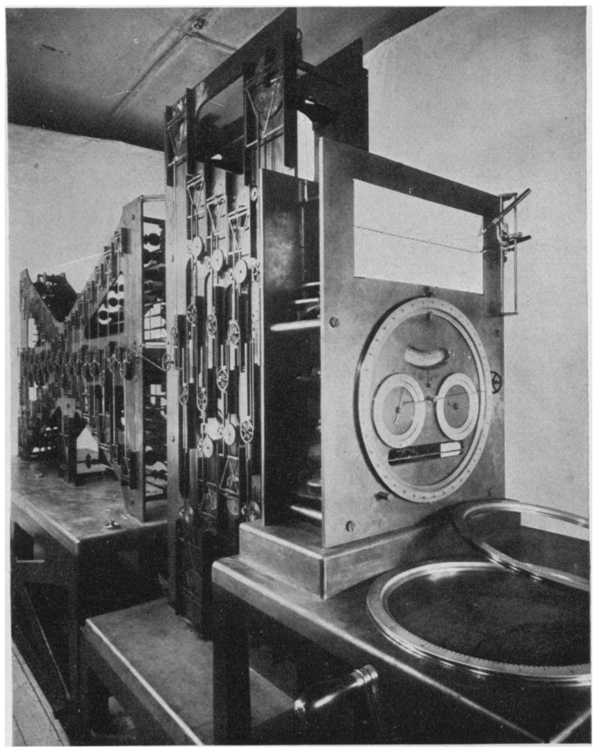

FIG. 2—Tide-predicting machine. On the machine the amplitudes of the constituent tides are set out on cranks, and the phases on dials. The height of tide due to any constituent tide is then given by the vertical motion of a slide on which is fixed a pulley with a chain working in the groove. This chain is made to travel over all the pulleys and thus sums instantly the constituent tides.

We cannot here enter into a detailed discussion of harmonic analysis. Unlike the nonharmonic method, which makes use of high and low waters only, the harmonic method makes use of the whole tidal wave, that is it uses hourly heights of the tide. The process, while not difficult, necessitates somewhat involved mathematical computations, out of which emerge the phases and amplitudes of the constituent tides sought. Though the theoretical number of these simple constituent tides is large, most of them are of such small magnitude that only between twenty and thirty need be computed to give tidal predictions of a high degree of precision.

Tide-Predicting Machines

Now it is obvious that the period, phase, and amplitude of a simple constituent tide being known, it is not a difficult matter to find the height of

tide due to that constituent at any given future time. Hence, to predict the tide that will occur at some future time, it is only necessary to add together the heights of the constituent tides at that time. The labor involved in doing this by ordinary methods of computation, however, is so great as to be prohibitive. In 1872 Lord Kelvin devised a machine which mechanically effects the summation of all the various tidal components and thus put tidal prediction by the harmonic method on a practical basis.

For the United States the tide tables until 1885 were predicted by the nonharmonic method; but in that year a tide-predicting machine, constructed on plans devised by William Ferrel, of the Coast and Geodetic Survey, and based on the harmonic method, was put into operation. This machine differed considerably from that devised by Lord Kelvin and was well adapted for predicting the semidaily type of tide. For more complicated tides the Ferrel machine did not work so well. In 1912 it was superseded by a new tide-predicting machine, devised and constructed in the office of the Coast and Geodetic Survey. A view of this machine is shown (Fig. 2). It has been described in the Survey publication entitled "Description of the U. S. Coast and Geodetic Survey Tide-Predicting Machine No. 2." [1] By the new Coast and Geodetic Survey tide-predicting machine a tide table giving the time and height of every high and low water for every day of the year for any port can be made in about ten hours.

Accuracy of Tidal Predictions

Obviously the value of a tide table depends on the closeness with which the predictions agree with the tides as they actually occur. It is evident, however, that absolute agreement is quite out of the question, for the times and heights of the tide are modified to a considerable extent by prevailing meteorological conditions. Heavy winds and sudden changes of atmospheric pressure will affect both time and height of tide; and, until the meteorologist is in a position to predict, a year or more in advance, the exact meteorological conditions that will prevail, the predicted tides in the tide tables cannot take account of such changes. Corrections based on local knowledge, however, may be applied to the predicted tides to allow for changes in wind and atmospheric pressure.

As a test of the accuracy of tidal predictions, observations and predictions at Portland, Me., and Seattle, Wash., were compared for the months of May and November, 1919. At Portland the tide is of the semidaily type and has a rise and fall varying from seven to thirteen feet. At Seattle the tide is of the mixed type with a rise and fall varying from less than two feet to more than sixteen feet.

At Portland for the month of May the greatest difference between the predicted and observed times of tide was four-tenths of an hour, or about twenty-five minutes. For the whole month 59 per cent of the predicted

[1] *U. S. Coast and Geodetic Survey Special Publ. No. 32*, Washington, D. C., 1915.

times differed by not more than one-tenth of an hour from the observed times of tide, and 94 per cent by not more than two-tenths of an hour. For November 74 per cent of the predicted times differed by not more than one-tenth of an hour, and 97 per cent by not more than two-tenths of an hour—the greatest difference again being four-tenths of an hour.

At Seattle for May 56 per cent of the predicted times of tide differed by not more than one-tenth of an hour from the observed times, 78 per cent by not more than two-tenths of an hour, and 99 per cent by not more than three-tenths. Here, too, the greatest difference was four-tenths of an hour. For November 47 per cent of the predictions differed from the observations by not more than one-tenth of an hour, 78 per cent by not more than two-tenths, 99 per cent by not more than three-tenths; while the greatest difference was four-tenths of an hour.

It should be stated that no corrections were applied to the predicted times of tide for changes in meteorological conditions. Furthermore, it is to be remembered that near the times of high and low water the height of the tide is changing very slowly and that it is not practicable to determine the time of the observed high or low water with a precision greater than that represented by a tenth of an hour. Therefore a difference between observations and predictions of two or three-tenths of an hour is of little consequence, especially for practical purposes.

The study of the tides has engaged the minds of many brilliant mathematicians since the time of Newton and still constitutes a fertile field for investigation. The outstanding problems do not relate to the determination of the tidal forces, for these are known; they are altogether of a hydrodynamical character and are concerned with the motions of existing bodies of water as influenced by known tidal forces. As regards the prediction of tides, however, in so far as this is based on previous observations, the problem may be considered solved.